

PERFORMANCE EVALUATION OF THREE EVOLUTIONARY ALGORITHMS FOR SELECTIVE HARMONIC ELIMINATION IN VOLTAGE SOURCE MULTILEVEL INVERTER

ADEYEMO, I. A, OJO, J. A. & ADEGBOLA, O. A

Department of Electronic & Electrical Engineering, Ladoko Akintola University of Technology,
PMB, Ogbomoso, Oyo State, Nigeria

ABSTRACT

In Selective Harmonic Elimination-Pulse Width Modulation (SHE-PWM) technique, optimal switching angles at fundamental switching frequency are computed such that low order harmonics are eliminated, while the fundamental voltage is obtained as desired. In this paper, ant colony optimization (ACO), particle swarm optimization (PSO), and real coded genetic algorithm (RCGA) were implemented and compared for solving selective harmonic elimination (SHE) equations of an 11-level inverter. Using the same population size and the same step size of modulation index, performance evaluations of the three methods show that PSO is the fastest, RCGA are is the most efficient in terms of low order harmonic elimination while ACO is the most efficient in terms of minimization of total harmonic distortion (THD) over a wide range of modulation indices. Computational results are validated with MATLAB simulations.

KEYWORDS: Multilevel Inverter, Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO), Real Coded Genetic Algorithm (RCGA)

INTRODUCTION

Multilevel power conversion is a rapidly growing area of power electronics with good potential for further development. The concept of utilizing multiple small voltage steps to perform power conversion was developed from the idea of step approximation of sinusoid [1]. The unique structure of multilevel converters enables the construction of relatively high power converters with improved harmonic spectrum using relatively low power semiconductor devices. This has resulted into the ability of the converters to meet stringent power quality and high power demands. There are several advantages to multilevel power conversion approach when compared with the traditional two-level power conversion. The smaller voltage steps yield lower switching losses, improved power quality, lower electro-magnetic interference (EMI), lower voltage change rate (dv/dt), and lower torque ripple [2], [3].

Harmonic elimination in multilevel converters has been the focus of intensive research for many decades. To improve converters performance and output power quality, several modulation techniques used in conventional two-level inverter have been modified and deployed in multilevel inverters. These include sinusoidal pulse width modulation (SPWM), selective harmonic elimination (SHE) method, space vector control (SVC), and space vector pulse width modulation (SVPWM) [4]. SHE method at fundamental switching frequency however, arguably gives the best result because of its high spectral performance and considerably reduced switching loss. Selective harmonic elimination (SHE) or programmed pulse width modulation scheme is a switching technique for inverters that provides direct control over the output waveform harmonics. In this method, the switching angles are chosen (programmed) to eliminate selected

harmonics while the fundamental harmonic is satisfied. The implementation of this technique involves solving S number of equations in order to eliminate $(S-1)$ selected low order harmonics. With the increasing number of equations, the inverter voltage waveform approaches a nearly sinusoidal waveform with low harmonic distortion. However, the major drawback of this approach is the heavy computational burden involved in solving the transcendental nonlinear equations known as SHE equations that characterize the harmonics.

Several methods that have been reported for solving SHE equations can be classified into two groups: The first group is based on deterministic approach using exact algorithms. Newton Raphson iterative method [5] is one of these. The main disadvantage of iterative methods is that they diverge if the arbitrarily chosen initial values are not sufficiently close to the roots. They also risk being trapped at local optima and fail to give all the possible solution sets. The theory of symmetric polynomials and resultants [6] has been proposed to determine the solutions of the SHE equations. A difficulty with this approach is that as the number of levels increases, the order of the polynomials becomes very high, thereby making the computations of solutions of these polynomial equations very complex. Another approach uses Walsh functions [7], [8], [9] where solving linear equations, instead of non-linear transcendental equations, optimizes the switching angle. The method results in a set of algebraic matrix equations and the calculation of the optimal switching angles is a complex and time-consuming operation.

The second group is based on probabilistic approach using heuristics that minimize rather than eliminate the selected harmonics. Population-based evolutionary algorithms (EAs) such as genetic algorithm [10], particle swarm optimization [11], ant colony system [12] and bee algorithm [13] have been reported for computing the switching angles that eliminate 5th and 7th harmonics in 7-level inverter. The main benefits of EAs are improved convergence and the ability to find multiple solution sets over a wide range of modulation indices. These can be attributed to the parallel nature of EAs i.e. a search through a population of solutions rather than a sequential search for individual solutions, as in iterative method. EAs are derivative free and are successful in locating the optimal solution, but they are usually slow in convergence and require much computing time.

MULTILEVEL INVERTER

- **Multilevel Inverter Topologies**

The three main multilevel inverter topologies are diode-clamped inverter which is based on neutral point converter [14], flying capacitor inverter [15], and cascaded H-bridge inverter [16]. With increasing number of levels, flying capacitor inverter becomes more difficult to realize because each capacitor has to be charged with different voltages while diode-clamped inverter suffers from DC link voltage unbalancing problem. Among the topologies, cascaded H-bridge inverter with separate DC sources requires the least number of components. Its modular structure and circuit layout flexibility make it suitable for high voltage and high power applications.

Cascaded H-bridge multilevel inverter (CMLI) consists of a number of H-bridge inverter units with separate DC source (SDCS) for each unit. The units are connected in series as shown in Figure 1 for an N-level inverter such that the synthesized output voltage waveform is the sum of all the individual H-bridge outputs. The number of output phase voltage levels in a cascaded H-Bridge inverter is given by $N=2S + 1$, where S is the number of cascaded H-bridges per phase.

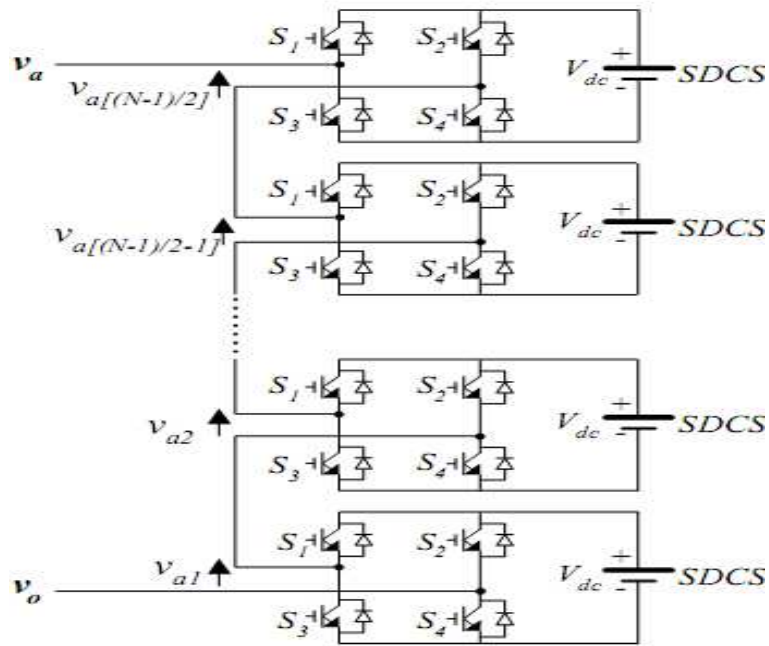


Figure 1: Single-Phase Structure of an N-Level Cascaded H-Bridge Multilevel Converter

Each H-bridge unit can produce three voltage levels: $+V_{dc}$, zero, and $-V_{dc}$ by different combinations of the four switches S_1 , S_2 , S_3 , and S_4 shown in the Figure1. To obtain $+V_{dc}$, switches S_1 and S_4 are turned on, whereas $-V_{dc}$ can be obtained by turning on switches S_2 and S_3 . By turning on S_1 and S_2 , or S_3 and S_4 , the output voltage is zero. By connecting a sufficient number of units in cascade and using an appropriate modulation scheme, a nearly sinusoidal voltage is produced. Shown in Figure 2 is the output phase voltage waveform of an 11-level inverter. The expression for the output phase voltage is given by

$$v_{an} = v_{a1} + v_{a2} + v_{a3} + v_{a4} + v_{a5} \tag{1}$$

• **Mathematical Model of SHE-PWM**

Generally, any periodic waveform such as the staircase waveform shown in Figure2 can be shown to be the superposition of a fundamental signal and a set of harmonic components. By applying Fourier transformation, these components can be extracted since the frequency of each harmonic component is an integral multiple of its fundamental [17].



Figure 2: Output Voltage Waveform of an 11-Level Inverter

Assuming a quarter wave symmetry and equal amplitude of all DC sources, the Fourier series expansion of the staircase output voltage waveform shown in Figure 2 is given by equation (2).

$$V(\omega t) = V_n(\alpha) \sin(n\omega t) \quad (2)$$

Where n is the harmonic order, V_n is the amplitude of n th harmonic, and the switching angles are constrained between zero and $\pi/2$. Due to odd quarter-wave symmetry; harmonics with even order become zero. Hence V_n is given by

$$V_n(\alpha) = \frac{4V_{dc}}{n\pi} \sum_{k=1}^s \cos(n\alpha_k), \text{ for odd } n \quad (3)$$

$$V_n(\alpha) = 0, \text{ for even } n \quad (4)$$

S in eqn. (3) is the number of switching angles. Combining equations (2), (3) and (4)

$$v(\alpha) = \sum_{n=1,3,5,\dots}^{\infty} \frac{4V_{dc}}{n\pi} (\cos(n\alpha_1) + \cos(n\alpha_2) + \dots + \cos(n\alpha_s)) \sin n\omega t \quad (5)$$

$$\text{Subject to } 0 < \alpha_1 < \alpha_2 < \dots < \alpha_s \leq \pi/2$$

The objective of SHE-PWM is to eliminate the lower order harmonics which are more harmful and more difficult to remove with filter while higher order harmonics are removed with low pass filter. In an 11-level inverter, there are five H-bridges per phase which translate into five degrees of freedom or switching angles. One degree of freedom is used to control the magnitude of the fundamental output voltage while the remaining four degrees of freedom are used to eliminate low order harmonics starting from 3rd order for single phase applications or from the 5th order for 3-phase applications equation. Ideally, given a desired fundamental voltage V_1 , the switching angles are determined such that (5) becomes

$$V(\omega t) = V_1 \sin(\omega t) \quad (6)$$

From equation (5), the expression for the fundamental output voltage V_1 in terms of the switching angles is given by

$$V_1 = \frac{4V_{dc}}{\pi} (\cos(\alpha_1) + \cos(\alpha_2) + \dots + \cos(\alpha_s)) \quad (7)$$

The relation between the fundamental voltage and the maximum obtainable fundamental voltage $V_{1\max}$ is given by modulation index. The modulation index, m_i , is defined as the ratio of the fundamental output voltage V_1 to the maximum obtainable fundamental voltage $V_{1\max}$. The maximum fundamental voltage is obtained when all the switching angles are zero [5]. From equation (7),

$$V_{1\max} = \frac{4SV_{dc}}{\pi} \quad (8)$$

$$\therefore m_i = \frac{V_1}{V_{1\max}} = \frac{\pi V_1}{4SV_{dc}}$$

Hence,

$$V_1 = m_i \left(\frac{4SV_{dc}}{\pi} \right) \quad \text{for } 0 < m_i \leq 1 \quad (9)$$

In balanced three-phase power system, the triplen harmonics in each phase need not be cancelled as they automatically cancel in the line-to-line voltages, and as a result only non-triplen odd harmonics are present in the line-to-line voltages. So, to satisfy fundamental harmonic and eliminate 5th, 7th, 11th, and 13th order harmonics that constitute the low order harmonics in an 11-level inverter, the appropriate modulation index and switching angles are computed by solving the transcendental nonlinear equations known as SHE equations that characterize the selected harmonics[5], [6]:

$$\begin{aligned} \frac{4V_{dc}}{\pi} (\cos(\alpha_1) + \cos(\alpha_2) + \dots + \cos(\alpha_5)) &= V_1 \\ \cos(5\alpha_1) + \cos(5\alpha_2) + \dots + \cos(5\alpha_5) &= V_5 \\ \cos(7\alpha_1) + \cos(7\alpha_2) + \dots + \cos(7\alpha_5) &= V_7 \\ \cos(11\alpha_1) + \cos(11\alpha_2) + \dots + \cos(11\alpha_5) &= V_{11} \\ \cos(13\alpha_1) + \cos(13\alpha_2) + \dots + \cos(13\alpha_5) &= V_{13} \end{aligned} \quad (10)$$

In eqn. (10), V_5 , V_7 , V_{11} , and V_{13} are set to zero to in order to eliminate 5th, 7th, 11th and 13th harmonics respectively. The correct solution must satisfy the condition

$$0 \leq \alpha_1 < \alpha_2 < \dots < \alpha_5 \leq \pi/2 \quad (11)$$

Equation (9) in equation (10) yields:

$$\begin{aligned} \cos(\alpha_1) + \cos(\alpha_2) + \dots + \cos(\alpha_5) &= 5m_i \cos(5\alpha_1) + \cos(5\alpha_2) + \dots + \cos(5\alpha_5) = 0 \\ \cos(7\alpha_1) + \cos(7\alpha_2) + \dots + \cos(7\alpha_5) &= 0 \\ \cos(11\alpha_1) + \cos(11\alpha_2) + \dots + \cos(11\alpha_5) &= 0 \\ \cos(13\alpha_1) + \cos(13\alpha_2) + \dots + \cos(13\alpha_5) &= 0 \end{aligned} \quad (12)$$

Generally equation (12) can be written as

$$F(\alpha) = B(m_i) \quad (13)$$

The Total Harmonic Distortion (THD) is computed as shown in equation (13):

$$THD = \sqrt{\sum_{i=5,7,11,13,\dots}^{49} \left(\frac{V_i}{V_1} \right)^2} \quad (13)$$

OPTIMIZATION TECHNIQUES

- **Ant Colony Optimization (ACO)**

The ant colony optimization is swarm intelligence based meta-heuristic algorithm that was inspired by the food foraging behavior of natural ants. It is a probabilistic technique for solving combinatorial optimization problems that can be reduced to finding good path through graphs [18]. ACO algorithm has been extended to solving continuous combinatorial optimization problems using a variety of ACO algorithm called variable sampling ant colony optimization (SamACO) algorithm [19]. SamACO algorithm offers an efficient incremental solution construction method based on the

sampled values. The basic idea behind SamACO algorithm is that a population of agents (artificial ants) incrementally constructs solution to the sampled combinatorial optimization problem.

The steps that are involved in the implementation of SamACO algorithm are as follows:

Initialization Step

The search space is bounded such that the decision variables (solution components) X_i has values $x_i \in [l_i, u_i]$, $i=1,2,\dots,S$, where l_i and u_i are the lower and upper bounds of the decision variables X_i respectively, and S is the number of decision variables. The initial values of the decision variables are randomly sampled in the feasible domain as follows:

$$x_i^{(j)} = l_i + \frac{u_i - l_i}{P} (j - 1 + \text{rand}_i^{(j)}) \quad (14)$$

Where P is the initial number of candidate values for each decision variable x_i^j , rand is a random number uniformly distributed within $[0, 1]$, $i=1, 2, \dots, S$ and $j=1, 2, \dots, P$. SamACO uses a population, of size P divided into two pools such that $P = m + \vartheta$, m is the number of ants, ϑ is the exploitation frequency which controls the number of values to be sampled in the neighborhood of the best-so-far solution per iteration,

For each decision variable X_i , there are k_i sampled values $x_i^{(1)}, x_i^{(2)}, \dots, x_i^{(k_i)}$ from the continuous domain $[l_i, u_i]$. Each solution component has associated a pheromone value, τ_i^j , $i=1,2,\dots,S$, $j=1,2,\dots,k_i$, and a component-pheromone matrix M can be generated. The pheromone value τ_i^j reflects the desirability of adding the component value x_i^j to the solution

$$M = \begin{bmatrix} \{x_1^{(1)}, \tau_1^{(1)}\} & \{x_2^{(1)}, \tau_2^{(1)}\} & \dots & \{x_s^{(1)}, \tau_s^{(1)}\} \\ \{x_1^{(2)}, \tau_1^{(2)}\} & \{x_2^{(2)}, \tau_2^{(2)}\} & \dots & \{x_s^{(2)}, \tau_s^{(2)}\} \\ \vdots & \vdots & \ddots & \vdots \\ \{x_1^{(k_i)}, \tau_1^{(k_i)}\} & \{x_2^{(k_i)}, \tau_2^{(k_i)}\} & \dots & \{x_s^{(k_i)}, \tau_s^{(k_i)}\} \end{bmatrix} \quad (15)$$

Transition

The transition of the ants from one position to another is partially probabilistic and partially deterministic. The transition rule favours either exploration or exploitation. An artificial ant k has a memory of the positions that it has already visited and the pheromone content at each location stored in a Tabu list T^k . The memory size of the Tabu list depends on the ant population size as well as the number of movement made by the ants. In general, if there are m ants making N movement, the size of the Tabu list is $(m \times N)$. The iteration index l_i^k of the variable value selected by ant k for the i^{th} variable is:

$$l_i^k = \begin{cases} 1 & \text{if } q \leq q_o \text{ and } j = j^* \\ 0 & \text{if } q \leq q_o \text{ and } j \neq j^* \\ L_i^{(k)} & \text{if } q > q_o \end{cases} \quad (16)$$

Where

$$j^* = \arg \max \{\tau_i^{(1)}, \tau_i^{(2)}, \dots, \tau_i^{(m)}\} \quad (17)$$

$i = 1, 2, \dots, s$, $k = 1, 2, \dots, m$, $q \in [0, 1]$ is a uniform random value, and $q_o \in [0, 1]$ is a threshold parameter that represents the relative preference for either exploitation or exploration.

Dynamic Exploitation

When $q \leq q_o$, the ant chooses exploitation in the neighborhood of the solution set with the highest pheromone value from the m solutions generated in the previous iteration. The dynamic exploitation is used as a local search method to fine-tune the best-so-far solution. A radius r_i confines the search in the neighborhood of the best-so-far solution $x^{(0)} = (x_1^{(0)}, x_2^{(0)}, \dots, x_s^{(0)})$ to the interval $[x_i^{(0)} - r_i, x_i^{(0)} + r_i]$, $i = 1, 2, \dots, S$. The values of the variables in the best-so-far solution set are randomly selected to be increased, unchanged or reduced as

$$\hat{x}_i = \begin{cases} \min(x_i^{(0)} + r_i \cdot \sigma_i, u_i), & 0 \leq q < \frac{1}{3} \\ x_i^{(0)}, & \frac{1}{3} \leq q < \frac{2}{3} \\ \max(x_i^{(0)} - r_i \cdot \sigma_i, l_i), & \frac{2}{3} \leq q < 1 \end{cases} \quad (18)$$

Where $\sigma \in [0, 1]$.

The best-so-far solution set is then updated using elitism and generational replacement. The new solution set $\hat{x} = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_s)$ is evaluated and used to replace the best-so-far solution set if there is an improvement in the result. The dynamic exploitation process is repeated for ϑ times, and the newly generated solution components are recorded as $x_i^{(j)}$, where $j = m + 1, m + 2, \dots, m + g_i$, $i = 1, 2, \dots, S$. The number of solution components generated during the dynamic exploitation process is denoted by g_i . The radii are adaptively extended or reduced based on the exploitation result. If the best solution set produced by the exploitation process is better than the prevailing best-so-far solution set, the radii will be extended. Otherwise, the radii will be reduced.

$$r_i \leftarrow \begin{cases} r_i \cdot v_e, & v_e > 1 \\ r_i \cdot v_r, & 0 < v_r \leq 1 \end{cases} \quad (19)$$

Where v_e and v_r are the radius extension rate and the radius reduction rate, respectively. The initial radius value is given by:

$$r_i = \frac{u_i - l_i}{2m} \quad (20)$$

Random Exploration

When $q > q_o$, the ant resort to probabilistic exploration to select a random index $L_i^{(k)} \in \{0, 1, \dots, m + g_i\}$. Moving from position i , the ant chooses its next position j among the positions that have not been visited yet according to the probability distribution given as follows:

$$p_i^{(j)} = \frac{\tau_i^{(j)}}{\sum_{u=0}^{m+g_i} \tau_i^{(u)}}, \quad j = 0, 1, \dots, m + g_i \quad (21)$$

The solution components of the worst x solution sets that are constructed by the ants in the previous iteration are discarded and each solution component is replaced by x new values generated by a random exploration process. If the worst solution sets are denoted by $x^{(m-x+1)}, x^{(m-x+2)}, \dots, x^{(m)}$. The new solution components for the solution set $x^{(j)}$ are randomly generated as follows:

$$x^{(j)} = l_i + (u_i - l_i) \cdot \text{rand}_i^{(j)} \quad (22)$$

Where $i = 1, 2, \dots, S$ and $j = (m - x + 1), (m - x + 2), \dots, m$.

Random exploration ensures diversity and prevents premature convergence to local minima.

Pheromone Update

Initially, each solution component is assigned an initial pheromone value τ_o . The pheromone values are updated based on the quality of solutions constructed by the ants. The update is biased towards the best solutions constructed by the ants such that ACO concentrates the search in the regions of high quality solutions. Similar to MAX-MIN ant system [20], the pheromone values in SamACO algorithm are bounded to the interval $[T_{\min}, T_{\max}]$; in this case $[0.1, 1]$.

The pheromones on the non-optimal paths are evaporated. The selected non-optimal solution components have their pheromones evaporated as

$$\tau_i^{(j)} \leftarrow (1 - \rho) \tau_i^{(j)} + \rho \tau_{\min} \quad \text{for } 0 < \rho < 1$$

$$i = 1, 2, \dots, S \quad \text{and } j = 1, 2, \dots, m \quad (23)$$

Where τ_{\min} is the predefined minimum pheromone value and ρ is the pheromone evaporation rate.

The pheromones on the near optimal paths are reinforced, thus influencing more ants to follow the paths and hopefully find better solutions. The solution components in the selected best Ψ solutions have their pheromone reinforced as

$$\tau_i^{(j)} \leftarrow (1 - \beta) \tau_i^{(j)} + \beta \tau_{\max} \quad \text{for } 0 < \beta < 1$$

$$i = 1, 2, \dots, S \quad \text{and } j = 1, 2, \dots, \Psi \quad (24)$$

Where τ_{\max} is the predefined maximum pheromone value, β is the pheromone reinforcement rate, and Ψ is the elitist number.

- **Particle Swarm Optimization (PSO)**

Particle swarm optimization is a swarm intelligence based algorithm that was inspired by the social behavior in a flock of birds or a school of fish. In PSO, an initial population of potential solutions to the optimization problem called particles is randomly generated. Each particle in a swarm searches for the best position in the search space, while the social behavior that is modeled in PSO guide the swarm to the optimal region. Each particle in the search space is assigned a

randomized position and velocity. During successive iteration, the current position of each particle in the swarm is evaluated with an objective function, and each particle keeps track of its coordinates in the problem space, which are associated with the best solution (fitness) that it has achieved so far.

Based on the fitness evaluation of all the particles, the best position so far of the i^{th} particle in a d -dimensional space is called personal best ($Pbest$), and is denoted by $P_i = [p_{i1}, p_{i2}, \dots, p_{id}]$, while the overall best position obtained so far by any particle in the swarm is called global best ($Gbest$), and is denoted by $P_g = [p_{g1}, p_{g2}, \dots, p_{gd}]$. This implies that each particle has a memory which enables it to update its current position and velocity according to the distance between its current position and $Pbest$, as well as the distance between its current position and $Gbest$. If the velocity and position vectors of the i^{th} particle at iteration k are represented as $V_i = [v_{i1}, v_{i2}, \dots, v_{id}]$ and $X_i = [x_{i1}, x_{i2}, \dots, x_{id}]$, respectively, then the velocity and position of the particle in the next iteration are determined as follows:

$$v_i(k+1) = wv_i(k) + c_1r_1[p_i(k) - x_i(k)] + c_2r_2[p_g(k) - x_i(k)] \quad (25)$$

$$x_i(k+1) = x_i(k) + v_i(k+1) \quad (26)$$

where w is the inertia weight parameter that provides the balance between global exploration and local exploitation capabilities of the particle, $v_i(k)$ is the velocity of the particle at iteration k ; $x_i(k)$ is the position the particle at iteration k ; c_1 and c_2 are constants known as cognitive and social coefficients, respectively; r_1 and r_2 are random values uniformly distributed within $[0, 1]$ [21].

The steps that are involved in the implementation of PSO algorithm are as follows:

- Randomly generate an initial population of particles subject to eqn. (11).
- Perform the fitness evaluation of the particles
- Update the personal best position $Pbest$ and global best position $Gbest$.
- Evaluate the velocity of each particle
- Compute new position of each particle using the updated velocity.
- Repeat the algorithm until any of the stopping criteria is met.

Real Coded Genetic Algorithm (RCGA)

Genetic algorithm (GA) is an evolutionary algorithm that was inspired by the study of genetics and survival of the fittest through the evolution mechanism observed in natural systems and population of living beings. Over successive generations, the parameters of a randomly created initial population of individuals, or potential solutions to the problem called strings or chromosomes are repeatedly modified by GA operators to create new (and hopefully better) population of solutions [22].

The steps that are involved in the implementation of RCGA algorithm are as follows:

Chromosome Representation

In an 11-level inverter, there are five switching angles which translate into five genes in a chromosome.

Each chromosome (potential solution) of the transcendental nonlinear equations is encoded as a real value numbers of the same length as the dimension of the search space.

Initialization

An initial population of chromosomes is randomly generated. The generated chromosomes are uniformly distributed between the lower and upper limits of the switching angles by satisfying eqn. (11)

Selection

GA begins the creation of new generation with the selection of chromosomes from the parent population based on their fitness evaluation. The fitness function is the function that is responsible for the evaluation of the solutions at each step

Crossover

Crossover operator is the main genetic operator, and it is applied with certain probability. Chromosome parts of selected parents are swapped to form new offspring for the next generation.

Mutation

In order to introduce diversity and prevent premature convergence, the genetic properties of the new offspring are deliberately altered with a low mutation probability.

The process of selection, crossover, and mutation is repeated until a maximum number of generations is reached or until the objective function has reached a preset value.

IMPLEMENTATION

Using MATLAB software, SamACO, PSO and RCGA algorithms were implemented to compute the optimal switching angles that eliminate 5th, 7th, 11th, and 13th harmonics in an 11-level inverter. In this work, the same population size of 40 was used for the implementation of the three algorithms, and the same number of iterations is 100. The genetic operators adopted for RCGA are tournament selection, heuristic crossover at the rate of 0.8, and dynamic or non-uniform mutation at the rate of 0.02. Solutions were computed for the three algorithms by incrementing the modulation index, m_i in steps of 0.001 from 0 to 1. A personal computer (2.66 GHz Intel Core i7 processor with 4GB Random Access Memory) running MATLAB R2014b on OS X Yosemite version 10.10 was used to carry out the computations.

The solution set at each step is evaluated with the fitness function. The objective here is to determine the switching angles such that the selected low order harmonics are either eliminated or minimized to an acceptable level while the fundamental voltage is obtained at a desired value. For each solution set, the fitness function is calculated as follows [12]:

$$f = \min_{\alpha_i} \left[\left(100 \frac{V_1^* - V_1}{V_1^*} \right)^4 + \sum_{s=2}^S \frac{1}{h_s} \left(50 \frac{V_{hs}}{V_1} \right)^2 \right]$$

$$i = 1, 2, \dots, S \quad (27)$$

Subject to eqn.(11)

Where V_1^* is the desired fundamental output voltage, S is the number of switching angles, h_s is the order of the s^{th} viable harmonic at the output of a three phase multilevel converter. For example, $h_2 = 5$, $h_4 = 11$. It should be noted that different weight are assigned to different harmonics in eqn. (25). Each harmonic ratio is weighted by inverse of its harmonic order, i.e. $1/h_s$. By this weighting method, higher importance is assigned to the low order harmonics, which are more harmful and difficulty to remove with filter.

In order to validate the observed analytical results, an 11-level single-phase Cascaded H-Bridge inverter was modelled in MATLAB-SIMULINK using SimPower System block set. In each of the five H-Bridges in the 11-level single-phase Cascaded H-Bridge inverter, 12V dc source is the SDCS, and the switching device used is Insulated Gate Bipolar Transistor (IGBT). Fundamental frequency switching scheme was adopted in this work because of its simplicity and low switching losses. Simulations were performed at the same arbitrarily chosen modulation index of 0.922 using solution sets previously calculated offline with each of the three algorithms. Fast Fourier Transform (FFT) analysis of the simulated phase voltage waveforms was performed to show the harmonic spectra of the synthesized waveforms and the corresponding THD value of each solution set was measured using the FFT block.

RESULTS AND DISCUSSIONS

Among the three algorithms, PSO is the fastest with average execution time of 453.02s, compared with SamACO and RCGA with average execution time of 1.94e+03s and 5.46e+03s, respectively. The plots of fitness function for each set of switching angles versus modulation index over the range of 0.1 to 1.0 are shown in Figure 3, Figure 6 and Figure 9 for SamACO, PSO, and RCGA, respectively. Solution sets with fitness value greater than 10^{-2} are rejected. When the fitness function at a modulation index is 10^{-2} or less, the corresponding switching angles are considered as a solution set.

Shown in Figure 4, Figure 7, and Figure 10 are the plots of switching angles that minimize 5th, 7th, 11th, and 13th harmonics in an 11-level inverter for SamACO, PSO, and RCGA, respectively. It can be seen from the figures that there are multiple solution sets at some modulation indices. In such cases, THD is computed for each of the multiple solution sets and the set with the least THD value is chosen and termed as a combined solution set.

As can be observed from the THD curves of the solution sets plotted in Figure 5, Figure 8, and Figure 11 for SamACO, PSO, and RCGA, respectively, the values of the 49th order THD are higher at lower modulation indices while they are considerably reduced at the upper end of modulation index. The plot of 13th order THD shows how efficiently the selected harmonics are minimized. Comparative study of Figure 5, Figure 8, and Figure 11 reveals that only SamACO finds multiple solution sets with 49th order THD that is less than 5% below modulation index of 0.85. However, 13th order THD are minimized rather than eliminated in most cases. None of the solution sets found with both PSO and RCGA below modulation index of 0.85 meets IEEE-519 standard. It should be noted that more solution sets are found with RCGA, and the selected low order harmonics are well attenuated in most cases.

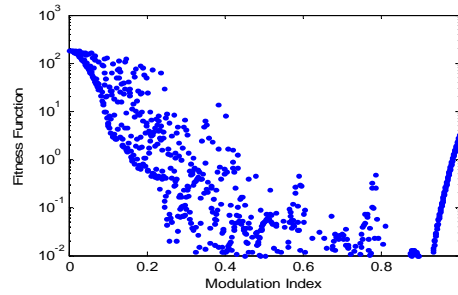


Figure 3: Fitness Function Versus Modulation Index for 11-Level CMLI Using Sam ACO

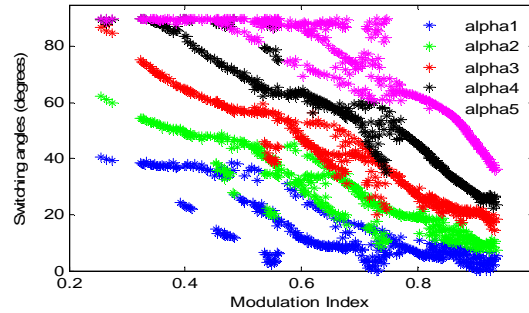


Figure 4: Switching Angles Versus Modulation Index for 11-Level CMLI Using Sam ACO

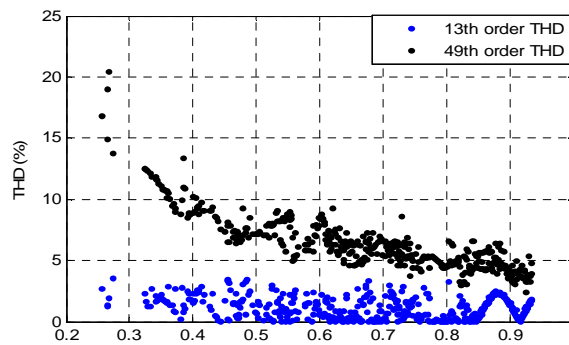


Figure 5: THD Versus Modulation Index for 11-Level CMLI Using Sam ACO

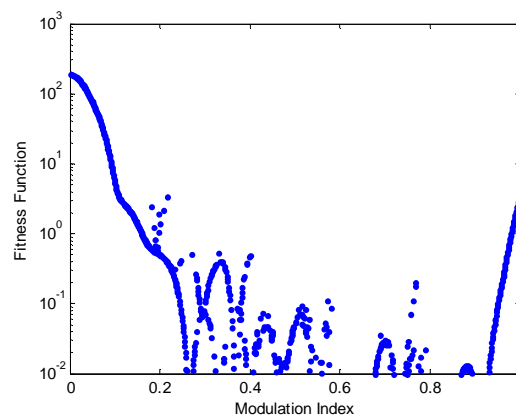


Figure 6: Fitness Function Versus Modulation Index for 11-Level CMLI Using PSO

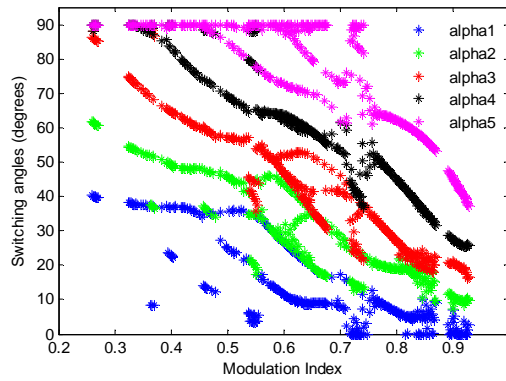


Figure 7: Switching Angles Versus Modulation Index for 11-Level CMLI Using PSO

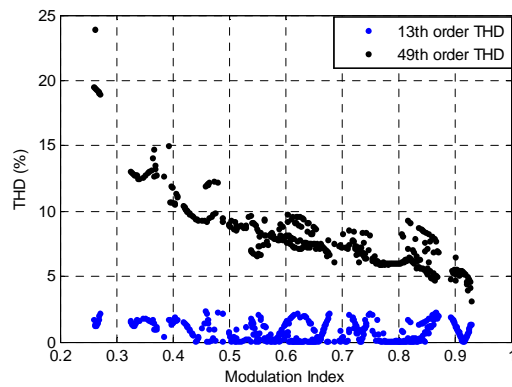


Figure 8: THD Versus Modulation Index for 11-Level CMLI Using PSO

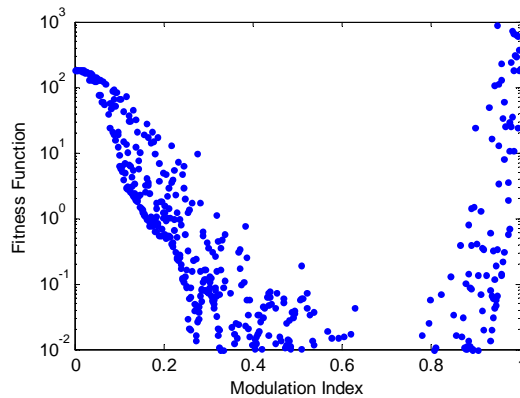


Figure 9: Fitness Function Versus Modulation Index for 11-Level CMLI Using RCGA

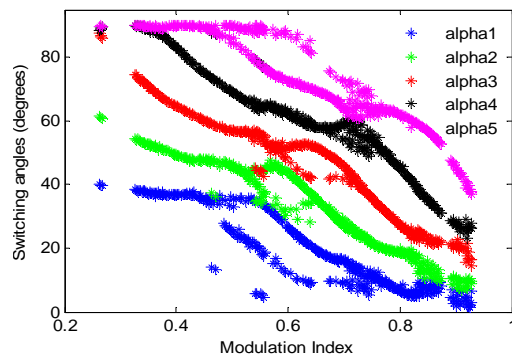


Figure 10: Switching Angles Versus Modulation Index for 11-Level CMLI Using RCGA

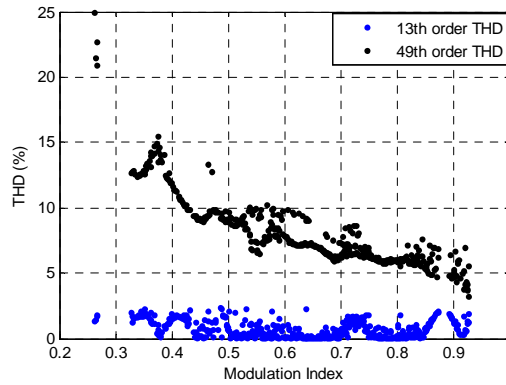


Figure 11: THD Versus Modulation Index for 11-Level CMLI Using RCGA

Shown in Table 1 are the values of the switching angles computed for the modulation index of 0.922 using SamACO, PSO and RCGA algorithms.

Table 1: Solution Sets at Modulation Index of 0.922

Optimizationn	Switching Angles at Modulation Index of 0.922				
	α_1	α_2	α_3	α_4	α_5
SamACO	6.550	7.100	17.371	27.139	39.178
PSO	0.000	9.680	18.588	25.499	39.766
RCGA	1.964	9.394	18.675	25.460	39.774

The analytically computed peak value of the fundamental output voltage given by eqn. (9) is $V_1 = m_i \left(\frac{4sV_{dc}}{\pi} \right) = 0.922 \left(\frac{4 \times 5 \times 12}{\pi} \right) = 70.41V_{(peak)}$ which closely agrees with simulation values of 70.35V, 70.35V, and 70.31V for SamACO, PSO, and RCGA, respectively.

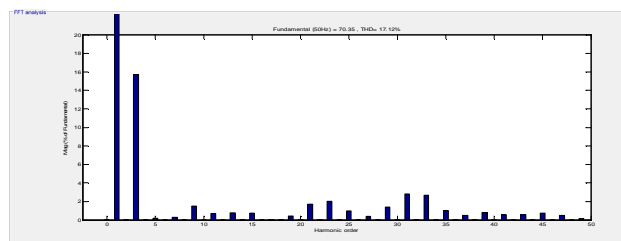


Figure 12: FFT Plot of Sam ACO Solution set at $m_i = 0.922$

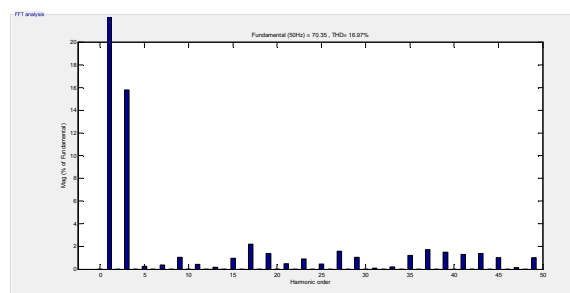


Figure 13: FFT Plot of PSO Solution Set at $m_i = 0.922$

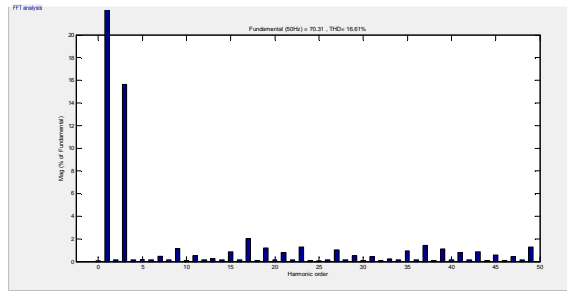


Figure 14: FFT Plot of RCGA Solution Set at $m_i = 0.922$

Shown in Table 2 is the comparative study of the analytical and simulation values of the 13th order THD of the three algorithms. From the table, it is apparent that PSO and RCGA are more efficient than Sam ACO in terms of the selected harmonics elimination.

Table 2: 13th Order thd (%) of the Simulated Voltage

Optimizationn	13 th Order THD (%)	
	Analytical Value	Simulation Value
SamACO	1.14	1.13
PSO	0.63	0.64
RCGA	0.63	0.74

The comparative study of the analytical and simulation values of the 49th order THD of the three algorithms is presented in Table 3.

Table 3: 49th Order Thd (%) of the Simulated Voltage

Optimizationn	49 th Order THD (%)	
	Analytical Value	Simulation Value
SamACO	4.28	4.29
PSO	5.02	4.30
RCGA	4.31	3.74

It can be seen from the tables that the simulation values closely agree with the analytical values. It should be noted that THD values of 17.12%, 16.97%, and 16.61% are shown in Figures 12, 13, and 14, respectively; the reason for this is that the THD values shown are for phase voltages which include triplen harmonic components while analytical values are for line voltages which exclude triplen harmonic components.

CONCLUSIONS

Three population-based algorithms with random initial values have been successfully implemented for solving the transcendental nonlinear equations characterizing the harmonics in an 11-level inverter. The algorithms are derivative-free, accurate and globally convergent. Performance evaluation of the three methods shows that PSO is the fastest; RCGA is most efficient in terms of low order harmonic elimination while SamACO is the most efficient in terms of THD minimization over a wide range of modulation indices. Analytically observed results are validated with simulations and both are in close agreement.

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